Lecture 21 - Pressure, stress, elastic moduli and density

A very broad classification of the states of matter is: Solid, liquid, gas. We all know examples of these states so for example at room temperature steel is a solid, water is liquid and oxygen is a gas. We also know the basic differences between these states of matter:

- Solids do not flow (they resist a shear force).
- Liquids flow but resist compression or expansion.
- Gases expand and fill any container they are put in.

During the next 3 weeks we will be studying the properties of solids liquids and gases. There are many subdivisions within the broad classes above, for example it is possible to make glasses which have an atomic structure like a liquid, but which do not flow at least on the time scale of a human being. These materials are called amorphous solids. There are also many different types of crystal structures for solids and the particular crystal structure plays a big role in determining the properties of the solid e.g. in determining whether the material is a conductor or an insulator. As we change from a solid to a liquid or from a liquid to a gas, properties such as density, elastic moduli, pressure, volume etc usually change. Before we can talk about these changes, we need to introduce each of these properties more precisely.

Pressure and stress

To get started we need to understand pressure and stress, which are better for describing the mechanical behavior of solids. Pressure is related to force through,

$$P = Pressure = Force/area \tag{1}$$

Stress is also force/area, but now the force is applied in different ways. Uniaxial tensile stress is a stress which tries to stretch a object and the shear stress tries to shear it. We then have,

$$Uniaxial\ stress = Tensile\ Force/Area \tag{2}$$

and

$$Shear\ stress = Shear\ Force/Area \tag{3}$$

Note that these forces are applied in a manner that ensures that the material sample does not translate or rotate. The unit for pressure or stress is the

Pascal (Pa) which is equal to N/m^2.

Consequences of applying pressure or stress

When we apply a compressive pressure, P, to a material, it usually gets smaller. In technical terms we say that pressure or compression leads a volume change. If its initial volume is V, then its volume change is ΔV. Every material is characterized by its compressibility and the variable which is used to describe this is called the bulk modulus (B). Squishy materials have a low bulk modulus and are easy to compress, while hard materials have a large bulk modulus. The precise relation between these quantities is,

$$P = -B\frac{\Delta V}{V} \tag{4}$$

Notice that the ΔV is negative because the volume decreases. Also, since $\Delta V/V$ is dimensionless, the bulk modulus has the same units as the pressure. The bulk modulus is one of the so called elastic moduli and the reason for this can be understood by considering the case of applying uniaxial tension to a piece of material. In that case we have,

$$\frac{F_{tensile}}{A} = Y\frac{\Delta L}{L_0} \tag{5}$$

In this case the piece of material of length L_0 extends by a distance ΔL when a tensile stress is applied to it. Now notice that this looks like the external force required to stretch an elastic spring,

$$F = kx \tag{6}$$

By comparing the two equations we can see that $k = YA/L_0$ is the effective spring constant of a material of cross-section A and of length L_0.

When a shear stress is applied to a piece of material, it leads to a shear distortion $\Delta x/h$

$$\frac{F_{shear}}{A} = S\frac{\Delta x}{h} \tag{7}$$

where S is the shear modulus.

Density

Finally we define the density, ρ which is the mass per unit volume,

$$\rho = M/V \tag{8}$$

The density of water at 4^0C is $10^3kg/m^3$ and the specific gravity of a material is the ratio of the density of the material to the density of water at 4^0C.